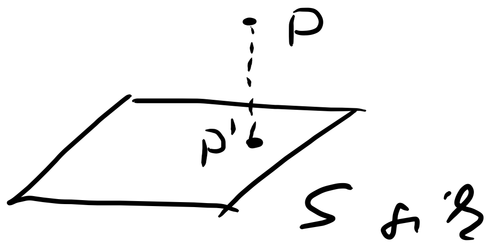
VII/6 $\quad W = \left\{ \begin{pmatrix} x\\ y\\ u\\ v \end{pmatrix} \in \mathbb{R}^4 \;\middle|\; x+y+u+v=0 \right\}$

„hipersík" = altér : dim = dim $V-1$

(vagy ennek eltoltja geom. ért.)

Pont távolsága ettől $(1,2,3,4)$.



P-t $\perp$-en vetítjük S-re

PP' távolság kell (≥ 0)

$\overrightarrow{PP'} \perp \forall$ S-beli vektorra.

S egyenlete $x+y+u+v=0$ $\quad \langle (1,1,1,1), v \rangle = x+y+u+v$.

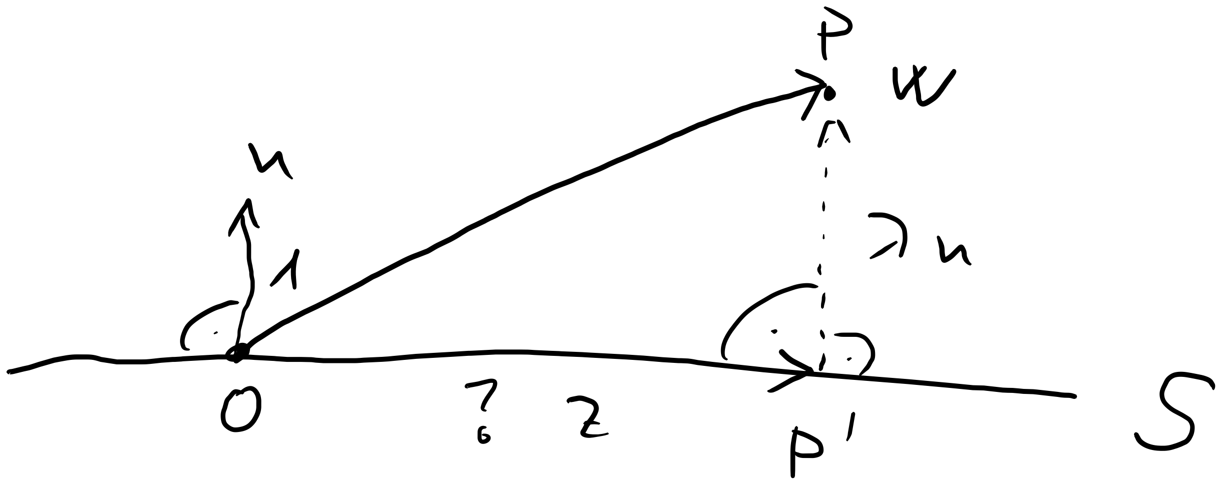
$(1,1,1,1) \perp v=(x,y,u,v)$

leolvasható az egyenletből, $\perp$ az S-re.

n = normálvektor : normálva $\frac{1}{2}(1,1,1,1)$.

$(1,2,3,4)$ távolsága S-től $\left| \langle (1,2,3,4), \frac{1}{2}(1,1,1,1) \rangle \right| =$

$= 10/2 = \underline{\underline{5}}$



$OP' \perp PP'$
$\in S$

$$\vec{OP} = \vec{OP'} + \vec{P'P}$$

$$v = z + \lambda u \qquad z = v - \lambda u \qquad z \perp u$$

$$0 = \langle z, u \rangle = \langle v - \lambda u, u \rangle = \langle v, u \rangle - \lambda \underbrace{\overbrace{\langle u, u \rangle}^{1}}_{=-\lambda}$$

$$\boxed{\langle v, u \rangle = \lambda}$$

PP' hossza =

= λu hossza = $|\lambda|$

u egységvektor.

A zérustól távolság $|\langle v, u \rangle|$

① $v = \begin{bmatrix} x \\ y \end{bmatrix}$ $\qquad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$v^T M v$

$= \langle v, Mv \rangle = ? = \bar{x}(ax+by) + \bar{y}(cx+dy) = (*)$

$Mv = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$ $\qquad Q(x,y)$ $\parallel$ kv. alak.

$(*) = a \underbrace{\bar{x} x}_{|x|^2} + b \bar{x} y + c \bar{y} x + d \underbrace{\bar{y} y}_{|y|^2}.$

$\mathbb{R}$: nincs konjugálás $\quad * = ax^2 + (b+c)xy + dy^2.$

$$\begin{array}{c|cc} & x & y \\ \hline x & a & b \\ y & c & d \end{array}$$

Szimm. mátrix kell.

$$\begin{bmatrix} a & \frac{b+c}{2} \\ \frac{b+c}{2} & d \end{bmatrix}.$$

$$2x^2 + 2xy + 2y^2 \quad M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\frac{x^2}{2} - xy + \frac{y^2}{2} + 3\frac{x^2}{2} + 3xy + 3\frac{y^2}{2}$$

Ell: ||

$$1\left(\frac{1}{\sqrt{2}}x + \frac{-1}{\sqrt{2}}y\right)^2 + 3\left(\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y\right)^2$$

$$\begin{vmatrix} 2-x & 1 \\ 1 & 2-x \end{vmatrix} = (2-x)^2 - 1 = x^2 - 4x + 4 - 1 = $$
$$= x^2 - 4x + 3$$

$(2-x)^2 = 1$

$2 - x = \pm 1$

gyökök $\dfrac{2 \pm \sqrt{16-12}}{2} = \begin{matrix} 1 \\ 3 \end{matrix}$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y \\ x+2y \end{bmatrix} = 1\begin{bmatrix} x \\ y \end{bmatrix} \quad 2x + y = x \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= 3\begin{bmatrix} x \\ y \end{bmatrix} \quad 2x + y = 3x \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

normální!

$\lambda = 1 \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 3 \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$1, 3 > 0$

$\Rightarrow$ Pozitív definit

$$2x^2 + 4xy + 2y^2 \qquad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

positiv semidef.

$$\begin{vmatrix} 2-x & 2 \\ 2 & 2-x \end{vmatrix} = (2-x)^2 - 4 = 0$$

$$2-x = \pm 2$$

$$x = 2 \mp 2 = \underline{\underline{0, 4}}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 2y \\ 2x + 2y \end{bmatrix} = 4 \cdot \begin{bmatrix} x \\ y \end{bmatrix} \qquad 2x + 2y = 4x \qquad \underline{\underline{y = x}}$$

$$= 0 \begin{bmatrix} x \\ y \end{bmatrix} \qquad 2x + 2y = 0 \qquad y = -x$$

$$\lambda = 4 \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 0 \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\underline{\underline{4\left(\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y\right)^2}} + 0 \cdot \text{neu ordnen!} \overset{\text{Ell.}}{=} 2(x+y)^2 \ \checkmark$$

VIII/3 $x^2+y^2+(x+y)^2 = (\sqrt{3}x)^2+(\sqrt{3}y)^2-(x-y)^2$

$\begin{pmatrix}0\\ \sqrt{3}\end{pmatrix}$ $\begin{pmatrix}1\\ -1\end{pmatrix}$ $\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix}$

poz. def.? indefinit ?!? ~~ROSSZ~~

tényleg poz. def. ✓

$\underset{>0}{\lambda_1}(k_1)^2 + \underset{>0}{\lambda_2}(k_2)^2 + \underset{<0}{\lambda_3}(k_3)^2$

Miért indefinit?

Pl x^2-y^2 indefinit volt.

Poz.: x valami, $y=0$

Neg: $x=0$, y valami.

k_1, k_2, k_3 kifejezésük ONB-ből számolandó.

$k_1 = \langle b_1, \begin{pmatrix}x\\y\\z\end{pmatrix}\rangle \Rightarrow$ F

$k_1 = a$
$k_2 = b$
$k_3 = c$

$\forall a, b, c$-re van M.O.!!

$2xy + 2yz + 2xz =$ négyzetösszeg alak.

$$\begin{array}{c|c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array} \qquad \left|\begin{matrix} -x & 1 & 1 \\ 1 & -x & 1 \\ 1 & 1 & -x \end{matrix}\right| \begin{matrix} -x & 1 \\ 1 & -x \\ 1 & 1 \end{matrix} =$$

$$= (-x)^3 + 1 + 1 + x + x + x = (-x)^3 + 3x + 2.$$

(Rac. gyökteszt): -1 gyök, 2 gyök
(Horner) -1 kétszeres
(VAN VIÈTE)

$\Rightarrow$ indefinit

$\lambda = 2$ egyszeres
$\lambda = -1$ kétszeres

s.é.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y+z \\ x+z \\ x+y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left.\begin{aligned} y + z &= 2x \\ x + z &= 2y \\ x + y &= 2z \end{aligned}\right\} \Rightarrow x = y = z \qquad \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} y+z \\ x+z \\ x+y \end{bmatrix} = -1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \left.\begin{aligned} y+z+x&=0 \\ x+z+y&=0 \\ x+y+z&=0 \end{aligned}\right\} \; W$$

$x+y+z=0$ 2-dim sajátaltér $\lambda=-1$-hez

GRAM-SCHMIDT

$b_1 = \frac{1}{\sqrt{2}}(1,-1,0)$

$v = (0,1,-1)$

$$v - \underbrace{\langle e_1, v\rangle}_{= \frac{1}{\sqrt{2}}\cdot(-1)} e_1 = (0,1,-1) + \frac{1}{2}(1,-1,0) =$$

$$\frac{1}{\sqrt{2}}\cdot(-1)\,\frac{1}{\sqrt{2}}(1,-1,0)$$

$= \left(\frac{1}{2}, \frac{1}{2}, -1\right)$ ← benne van az altérben mert $\frac{1}{2}+\frac{1}{2}-1=0$ ✓

$\perp$ b_1-re, mert $\frac{1}{2}\cdot 1 + \frac{1}{2}(-1) + 0\cdot(-1) = 0$.

$(1/2, 1/2, -1) \rightsquigarrow (1, 1, -2)$ normálni

$$\left.\begin{array}{l} b_2 = \frac{1}{\sqrt{6}}(1, 1, -2) \\ b_1 = \frac{1}{\sqrt{2}}(1, -1) \end{array}\right\}$$

$\lambda = -1$ - hez.

$W: x + y + z = 0$

$\lambda = 2$ - höz $\frac{1}{\sqrt{3}}(1, 1, 1)$

$$Q = 2\left(\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z\right)^2 - $$
$$- \left(\frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y\right)^2 - \left(\frac{1}{\sqrt{6}}x + \frac{1}{\sqrt{6}}y - \frac{2}{\sqrt{6}}z\right)^2$$